6.6

a) $5\sqrt{27}\cdot 4\sqrt{6} = 20\sqrt{3^3\cdot 3\cdot 2} = 20\sqrt{3^4\cdot 2} = 20\times 3^2\sqrt{2} = 180\sqrt{2}$

c) $5\sqrt{\frac{3}{4}}\cdot\sqrt{\frac{8}{27}} = 5\sqrt{\frac{3\cdot 2^3}{2^2\cdot 3^3}} = 5\sqrt{\frac{2}{3^2}} = \frac{5\sqrt{2}}{3}$

e) $\sqrt{x}\,\sqrt[3]{x^2y^4} = \sqrt[6]{x^3(x^2y^4)^2} = \sqrt[6]{x^7y^8} = \sqrt[6]{x^6\cdot x\cdot y^6\cdot y^2} = xy\sqrt[6]{xy^2}$

f) $\sqrt[5]{\frac{x^2y^2}{z^2}} : \sqrt{\frac{xy}{z}} = \sqrt[10]{\left(\frac{x^2y^2}{z^2}\right)^2\cdot\left(\frac{z}{xy}\right)^5} = \sqrt[10]{\frac{x^4y^4z^5}{z^4x^5y^5}} = \sqrt[10]{\frac{z}{xy}}$

g) $\frac{5\sqrt{32}+7\sqrt{2}}{11\sqrt{8}} = \frac{5\sqrt{2^5}+7\sqrt{2}}{11\sqrt{2^3}} = \frac{5\times\sqrt{2^4\cdot 2}+7\sqrt{2}}{11\sqrt{2^2\cdot 2}} = \frac{20\sqrt{2}+7\sqrt{2}}{22\sqrt{2}} = \frac{27\cancel{\sqrt{2}}}{22\cancel{\sqrt{2}}} = \frac{27}{22}$

h) $\frac{\sqrt[9]{64a^3b^6}}{\sqrt[3]{2^2a}} = \frac{\sqrt[9]{2^6a^3b^6}}{\sqrt[3]{2^2a}} = \frac{\sqrt[3]{2^2ab^2}}{\sqrt[3]{2^2a}} = \sqrt[3]{\frac{\cancel{2^2}\cancel{a}b^2}{\cancel{2^2}\cancel{a}}} = \sqrt[3]{b^2}$

i) $\frac{\sqrt{\frac{ax^2-a}{a+b}}}{\sqrt{\frac{x-1}{a^2+ab}}} = \sqrt{\frac{a(x^2-1)}{a+b} : \frac{x-1}{a(a+b)}} = \sqrt{\frac{a(x^2-1)\,a\cancel{(a+b)}}{\cancel{(a+b)}(x-1)}} =$

$= \sqrt{\frac{a^2(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}} = a\sqrt{(x+1)}$

j) $\frac{\sqrt[4]{\frac{(x-1)^3}{(x+1)^5}(x^2-1)(x-1)^2}}{\sqrt{\frac{(x-1)^3}{(x+1)^2}}} = \frac{\sqrt[4]{\frac{(x-1)^5\cdot(x-1)(x+1)}{(x+1)^5}}}{\sqrt[4]{\left(\frac{(x-1)^3}{(x+1)^2}\right)^2}} = \frac{\sqrt[4]{\frac{(x-1)^6}{(x+1)^4}}}{\sqrt[4]{\frac{(x-1)^6}{(x+1)^4}}} = 1$

6.7

a) $\left(\sqrt[3]{25}\right)^4 = \sqrt[3]{\underbrace{25^4}_{(5^2)^4}} = \sqrt[3]{5^8} = \sqrt[3]{5^6\cdot 5^2} = 5^2\sqrt[3]{5^2} = \underline{25\sqrt[3]{25}}$

b) $\left(\sqrt[3]{x^5}\right)^6 = \sqrt[3]{x^{30}} = \underline{x^{10}}$

e) $\left(\sqrt[3]{(3^3a^4)}\right)^2 = \sqrt[3]{3^6\cdot a^8} = \sqrt[3]{3^6 a^6 a^2} = \underline{3^2a^2\sqrt[3]{a^2}} = \underline{9a^2\sqrt[3]{a^2}}$

f) $(\sqrt{7}+\sqrt{3})^2\cdot(5-\sqrt{21}) = \left[(\sqrt{7})^2+2\sqrt{7}\sqrt{3}+(\sqrt{3})^2\right]\cdot(5-\sqrt{21}) =$

$= (\underline{7}+2\sqrt{21}+\underline{3})(5-\sqrt{21}) = (10+2\sqrt{21})\cdot(5-\sqrt{21}) = 50-10\sqrt{21}+10\sqrt{21}-2(\sqrt{21})^2 =$

$= 50-\cancel{10\sqrt{21}}+\cancel{10\sqrt{21}}-42 = \underline{8}$

g) $(\sqrt{3}-\sqrt{2})^2 = (\sqrt{3})^2-2\sqrt{3}\sqrt{2}+(\sqrt{2})^2 = \underline{3}-2\sqrt{6}+\underline{2} = \underline{5-2\sqrt{6}}$

h) $(\sqrt{3}+\sqrt{2})^2 = (\sqrt{3})^2+2\sqrt{3}\sqrt{2}+(\sqrt{2})^2 = \underline{3}+2\sqrt{6}+\underline{2} = \underline{5+2\sqrt{6}}$

j) $(5\sqrt{3}-3\sqrt{5})^2 = (5\sqrt{3})^2-2\cdot5\sqrt{3}\cdot3\sqrt{5}+(3\sqrt{5})^2 = 25\times3-30\sqrt{15}+9\times5 =$

$= \underline{75}-30\sqrt{15}+\underline{45} = \underline{120-30\sqrt{15}} = \underline{30\cdot(4-\sqrt{15})}$

l) $\sqrt{\sqrt[3]{5}} = \underline{\sqrt[6]{5}}$

n) $\sqrt[3]{\sqrt{\sqrt{8}}} = \underline{\sqrt[12]{8}}$

ñ) $\sqrt{2\sqrt{2\sqrt{2}}} = \sqrt{\sqrt{2^2\cdot2\sqrt{2}}} = \sqrt{\sqrt{\sqrt{(2^3)^2\cdot2}}} =$

$= \sqrt[8]{2^6\cdot2} = \underline{\sqrt[8]{2^7}}$

o) $\sqrt[4]{\frac{25}{9}\sqrt[3]{\frac{9}{25}}} = \sqrt[4]{\sqrt[3]{\left(\left(\frac{5}{3}\right)^2\right)^3\cdot\left(\frac{3}{5}\right)^2}} = \sqrt[12]{\frac{5^6\cdot3^2}{3^6\cdot5^2}} = \sqrt[12]{\frac{5^4}{3^4}} = \underline{\sqrt[3]{\frac{5}{3}}}$

p) $\left(\sqrt[3]{\sqrt[7]{\sqrt{8x^3}}}\right)^7 = \sqrt[42]{(2^3x^3)^7} = \sqrt[42]{(2x)^{21}} = \underline{\sqrt{2x}}$

q) $\sqrt[4]{\frac{x}{y}\sqrt[3]{\frac{y}{x}}} = \sqrt[4]{\sqrt[3]{\frac{x^3}{y^3}\cdot\frac{y}{x}}} = \sqrt[12]{\frac{x^2}{y^2}} = \underline{\sqrt[6]{\frac{x}{y}}}$

6.8. RACIONALIZACIÓN

c) $\dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}$

d) $\dfrac{1}{\sqrt{27}}=\dfrac{1}{\sqrt{3^3}}=\dfrac{1}{3\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{3}}{9}$ $\quad 3$

f) $\dfrac{1}{\sqrt[3]{3}}\cdot\dfrac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}=\dfrac{\sqrt[3]{3^2}}{3}$

g) $\dfrac{3\sqrt{2}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{3\sqrt{6}}{3}=\sqrt{6}$

h) $\dfrac{12}{\sqrt{8}}=\dfrac{12}{2\sqrt{2}}=\dfrac{6}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{6\sqrt{2}}{2}=3\sqrt{2}$

i) $\dfrac{3a\sqrt{b}}{\sqrt{a}}\cdot\dfrac{\sqrt{a}}{\sqrt{a}}=\dfrac{3a\sqrt{a\cdot b}}{a}=3\sqrt{ab}$

$128|2$
$64|2^6$

j) $\dfrac{10}{\sqrt[5]{128}}=\dfrac{10}{\sqrt[5]{2^7}}=\dfrac{10}{2\sqrt[5]{2^2}}=\dfrac{5}{\sqrt[5]{2^2}}\cdot\dfrac{\sqrt[5]{2^3}}{\sqrt[5]{2^3}}=\dfrac{5\cdot\sqrt[5]{8}}{2}$

l) $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\cdot\dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}=\dfrac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\dfrac{(\sqrt{3})^2+2\sqrt{3}\sqrt{2}+(\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}=\dfrac{3+2\sqrt{6}+2}{3-2}=$

$=5+2\sqrt{6}$

m) $\dfrac{\sqrt{3}+\sqrt{2}}{1+\sqrt{3}}\cdot\dfrac{(1-\sqrt{3})}{(1-\sqrt{3})}=\dfrac{(\sqrt{3}+\sqrt{2})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}=$

$=\dfrac{\sqrt{3}-(\sqrt{3})^2+\sqrt{2}-\sqrt{2}\sqrt{3}}{1-(\sqrt{3})^2}=\dfrac{\sqrt{3}-3+\sqrt{2}-\sqrt{6}}{1-3}=\dfrac{3-\sqrt{2}-\sqrt{3}+\sqrt{6}}{2}$

n) $\dfrac{3\sqrt{5}-4}{\sqrt{5}-2}\cdot\dfrac{(\sqrt{5}+2)}{(\sqrt{5}+2)}=\dfrac{(3\sqrt{5}-4)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}=\dfrac{3(\sqrt{5})^2+6\sqrt{5}-4\sqrt{5}-8}{(\sqrt{5})^2-2^2}=$

$=\dfrac{15+6\sqrt{5}-4\sqrt{5}-8}{5-4}=7+2\sqrt{5}$

ñ) $\frac{1}{3(\sqrt{5}-\sqrt{2})} \times \frac{(\sqrt{5}+\sqrt{2})}{(\sqrt{5}+\sqrt{2})} = \frac{\sqrt{5}+\sqrt{2}}{3(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{\sqrt{5}+\sqrt{2}}{3.[(\sqrt{5})^2-(\sqrt{2})^2]} =$

$= \frac{\sqrt{5}+\sqrt{2}}{3(5-2)} = \frac{\sqrt{5}+\sqrt{2}}{9}$

⊛ $\frac{1}{3\sqrt{5}+\sqrt{2}} \cdot \frac{3\sqrt{5}+\sqrt{2}}{3\sqrt{5}+\sqrt{2}} = \frac{3\sqrt{5}+\sqrt{2}}{(3\sqrt{5}-\sqrt{2})(3\sqrt{5}+\sqrt{2})} = \frac{3\sqrt{5}+\sqrt{2}}{(3\sqrt{5})^2-(\sqrt{2})^2} =$

$= \frac{3\sqrt{5}+\sqrt{2}}{9 \times 5 - 2} = \frac{3\sqrt{5}+\sqrt{2}}{43}$

o) $\frac{\sqrt{2}-1}{\sqrt{8}-\sqrt{2}} = \frac{\sqrt{2}-1}{2\sqrt{2}-\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{(\sqrt{2}-1).\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$

p) $\frac{1-\frac{\sqrt{2}}{4}}{1+\frac{\sqrt{2}}{4}} + \frac{4\sqrt{2}}{7} = \frac{\frac{4-\sqrt{2}}{4}}{\frac{4+\sqrt{2}}{4}} + \frac{4\sqrt{2}}{7} =$

$= \frac{\cancel{4}.(4-\sqrt{2})}{\cancel{4}.(4+\sqrt{2})} + \frac{4\sqrt{2}}{7} = \frac{4-\sqrt{2}}{4+\sqrt{2}} + \frac{4\sqrt{2}}{7} = \frac{9-4\sqrt{2}}{7} + \frac{4\sqrt{2}}{7} =$ ⊛

$\frac{4-\sqrt{2}}{4+\sqrt{2}} \times \frac{4-\sqrt{2}}{4-\sqrt{2}} = \frac{(4-\sqrt{2})^2}{16-2} = \frac{16-8\sqrt{2}+2}{14} = \frac{18-8\sqrt{2}}{14} = \frac{\cancel{2}(9-4\sqrt{2})}{\cancel{2}\times 7}$

⊛ $= \frac{9-\cancel{4\sqrt{2}}+\cancel{4\sqrt{2}}}{7} = \frac{9}{7}$

(62 copias)

RADICALES 4º ESO. (FOTOCOPIA).

1) a) $= \sqrt[5]{\left(\frac{6zy^2}{x^2}\right)^5 \cdot \frac{3x^3y^2}{2z^4}} = \sqrt[5]{\frac{3^5 \cdot 2^5 \cdot z^5 y^{10} \cdot 3 \cdot x^3 \cdot y^2}{x^{10} \cdot 2 \cdot z^4}} = \sqrt[5]{\frac{3^6 \cdot 2^4 \cdot z \cdot y^{12}}{x^7}}$

b) $4\sqrt{2} \cdot \sqrt{2\sqrt{2}} \cdot \sqrt{2\sqrt[3]{2}} = 4\sqrt{2} \cdot \sqrt{\sqrt{2^2 \cdot 2}} \cdot \sqrt{\sqrt[3]{2^3 \cdot 2}} = 4\sqrt{2} \cdot \sqrt[4]{2^3} \cdot \sqrt[6]{2^4} =$

$= 4\sqrt[12]{2^6 \cdot 2^9 \cdot 2^8} = 4\sqrt[12]{2^{23}} = \sqrt[12]{(2^2)^{12} \cdot 2^{23}} = \sqrt[12]{2^{24} \cdot 2^{23}} = \sqrt[12]{2^{47}}$

c) $\frac{\sqrt[3]{3\sqrt{3}}}{\sqrt{3\sqrt[3]{3}}} = \frac{\sqrt[3]{\sqrt{3^2 \cdot 3}}}{\sqrt{\sqrt[3]{3^3 \cdot 3}}} = \frac{\sqrt[6]{3^3}}{\sqrt[6]{3^4}} = \sqrt[6]{\frac{3^3}{3^4}} = \sqrt[6]{\frac{1}{3}}$

2) a) $x^2 = 49 \Rightarrow x = \pm\sqrt{49} = \pm 7$

b) $x^2 = -49 \Rightarrow x = \pm\sqrt{-49} \notin \mathbb{R}$

c) $x^2 - 1 = 0 \Rightarrow x^2 = 1$; $x = \pm\sqrt{1} = \pm 1$

d) $x^2 + 1 = 0 \Rightarrow x^2 = -1$; $x = \pm\sqrt{-1} \notin \mathbb{R}$

e) $x^2 = a$ { si $a > 0 \Rightarrow x_1, x_2$: dos soluciones (opuestas)
si $a < 0 \Rightarrow$ No tiene soluciones en el números $\mathbb{R}$.

3) a) $-4^{1/2} = -\sqrt{4}$ $(= -2)$

b) $(-4)^{1/2} = \sqrt{-4}$ $(= \notin \mathbb{R})$

c) $(a^2+b^2)^{1/2} = \sqrt{a^2+b^2}$ $(\neq a+b$!!)

d) $(a^2b^2)^{1/2} = \sqrt{a^2b^2}$ $(= ab)$

e) $(2x^3y)^{2/5} = \sqrt[5]{(2x^3y)^2} =$
$= \sqrt[5]{2^2x^6y^2} = x\sqrt[5]{2^2xy^2}$

f) $(-32)^{-1/5} = \sqrt[5]{(-32)^{-1}} =$
$= \sqrt[5]{\frac{1}{-32}} = \sqrt[5]{\frac{1}{(-2)^5}} = \frac{1}{-2} = -1/2$

g) $2x^{3/5} = 2\sqrt[5]{x^3}$

h) $(2x)^{3/5} = \sqrt[5]{2^3x^3}$

4) a) $(x^{3/3} = x)$

$\sqrt[4]{\frac{\sqrt[3]{x^3}}{y^2}} \cdot \sqrt{\frac{\sqrt[3]{y}}{\sqrt[3]{x^2}}} = \sqrt[4]{\frac{x}{y^2}} \cdot \sqrt{\sqrt[3]{\frac{y}{x^2}}} = \sqrt[4]{\frac{x}{y^2}} \cdot \sqrt[6]{\frac{y}{x^2}} =$

$= \sqrt[12]{\frac{x^3}{y^6} \cdot \frac{y^2}{x^4}} = \sqrt[12]{\frac{1}{xy^4}}$

b) $\dfrac{\sqrt{3xy^3\sqrt{2x^2y}}}{\sqrt{6x^3y^4}} = \dfrac{\sqrt{\sqrt{3^2x^2y^6\cdot 2\cdot x^2y}}}{\sqrt{3\cdot 2\cdot x^3y^4}} = \dfrac{\sqrt[4]{3^2\cdot 2\cdot x^4y^7}}{\sqrt{3\cdot 2\cdot x^3y^4}} = \dfrac{\cancel{x}y\sqrt[4]{3^2\cdot 2\cdot y^3}}{\cancel{x}y^2\sqrt{3\cdot 2\cdot x}} =$

$= \dfrac{1}{y}\sqrt[4]{\dfrac{3^2\cdot 2\cdot y^3}{3^2\cdot 2^2\cdot x^2}} = \dfrac{1}{y}\sqrt[4]{\dfrac{y^3}{2x^2}}$

c) $\sqrt[4]{\sqrt[3]{x\sqrt{x}}} = \sqrt[4]{\sqrt[3]{\sqrt{x^2\cdot x}}} = \sqrt[24]{x^3} = \underline{\underline{\sqrt[8]{x}}}$

d) $(4\sqrt{x}-\sqrt{y})(\sqrt{x}+2\sqrt{y}) = 4\sqrt{x}\sqrt{x} + 8\sqrt{x}\sqrt{y} - \sqrt{y}\sqrt{x} - 2\sqrt{y}\sqrt{y} =$

$= 4x + 8\sqrt{xy} - \sqrt{xy} - 2y =$

$= 4x - 2y + 7\sqrt{xy}$

e) $\dfrac{1}{2}\sqrt{8} - \sqrt[4]{2^2} + \sqrt{\dfrac{2}{25}} = \dfrac{1}{\cancel{2}}\cancel{2}\sqrt{2} - \sqrt{2} + \dfrac{\sqrt{2}}{5} = \left(\cancel{1} - \cancel{1} + \dfrac{1}{5}\right)\sqrt{2} = \dfrac{1}{5}\sqrt{2}$

$= \cancel{\sqrt{2}} - \cancel{\sqrt{2}} + \dfrac{\sqrt{2}}{5} = \dfrac{\sqrt{2}}{5}$

✓ f) $(\sqrt{2}-\sqrt{3})(\sqrt{2}-\sqrt{3}) = (\sqrt{2}-\sqrt{3})^2 = (\sqrt{2})^2 - 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2 = 2 - 2\sqrt{6} + 3 =$

$= \underline{5 - 2\sqrt{6}}$

✓ 5) $A = \dfrac{2}{3}B \rightarrow A = \dfrac{2}{3}B$

$A + B = \underline{\underline{30}} \rightarrow A = 30 - B$

$\dfrac{2}{3}B = 30 - B \rightarrow \dfrac{2}{3}B + B = 30 \rightarrow$

$\rightarrow \dfrac{2B+3B}{3} = 30 \rightarrow \dfrac{5B}{3} = 30 \rightarrow B = \dfrac{90}{5} = 18$

$\dfrac{(\sqrt{98}-\sqrt{18})\cdot 30\sqrt{3}}{\sqrt{96}} = \dfrac{(7\sqrt{2}-3\sqrt{2})\cdot 30\sqrt{3}}{4\sqrt{6}} = \dfrac{4\sqrt{2}\cdot 30\sqrt{3}}{4\sqrt{6}} = \dfrac{120\sqrt{6}}{4\sqrt{6}} = \underline{\underline{30}}$

$\sqrt{98} = \sqrt{7^2\cdot 2} = 7\sqrt{2}$

$\sqrt{18} = 3\sqrt{2}$

$\sqrt{96} = \sqrt{2^5\cdot 3} = 2^2\sqrt{6}$

$\boxed{\begin{array}{l} A = 12 \text{ alumnes} \\ B = 18 \text{ alumnes} \end{array}}$